**Math 120 Intermediate Algebra**

**Applications Involving Quadratic Equations**

**Ex 1 (#6)** A turbo-jet flies 50 mph faster than a super-prop plane. If a turbo-jet goes 2000 mi in 3 hr less than it takes the super-prop to go 2800 mi, find the speed of each plane.

\[
\begin{align*}
&\text{Jet: } 2000, \ r+50, \ t-3 \\
&\text{Plane: } 2800, \ r, \ t
\end{align*}
\]

Let \( t \) = time (min) \( r \) = speed of plane \( t \) = time (min) \( r \) = speed of plane

\[
\begin{align*}
2000 &= (r+50)(t-3) \\
2000 &= 2800 \Rightarrow r &= \frac{2800}{t} \\
2000 &= r(t+50)(t-3) \\
2000 &= 280(t+50) \Rightarrow r &= \frac{650 \pm \sqrt{22500-4(3)(140,000)}}{12} \\
280 &= 12t + 24 \\
2800 &= 12t \\
\Rightarrow t &= \frac{2800}{12} = 233.33
\end{align*}
\]

**Ex 2 (#12)** Two pipes are connected to the same tank. Working together, they can fill the tank in 4 hr. The larger pipe, working alone, can fill the tank in 6 hr less than the smaller one. How long would the smaller one take, working alone, to fill the tank?

\[
\begin{align*}
\text{Jet Speed: } 400 \text{ mph} & \quad \text{Pipe Speed: } 350 \text{ mph} \\
\text{Let } t &= \text{time it takes smaller pipe to fill the tank} \\
\text{Let } t-6 &= \text{time it takes larger pipe to fill the tank} \\
\frac{t}{t-6} &= \frac{1}{4} \\
\frac{t}{t-6} &= \frac{1}{4} \\
4t &= t-6 \\
3t &= -6 \\
\Rightarrow t &= -2
\end{align*}
\]

**Ex 3 (#18)** Solve for \( k \): \( N = \frac{k^2-3k}{2} \)

\[
\begin{align*}
2N &= k^2-3k \\
k^2-3k-2N &= 0 \\
k &= \frac{3 \pm \sqrt{9+8N}}{2} \\
\text{Use quadratic formula} \quad a = 1, b = -3, c = -2N
\end{align*}
\]

**Ex 4 (#20)** Solve for \( s \): \( N = \frac{kq_1q_2}{s^2} \)

\[
\begin{align*}
N s^2 &= kq_1q_2 \\
S &= \sqrt{\frac{kq_1q_2}{N}} \\
S &= \frac{\sqrt{kq_1q_2}}{\sqrt{N}} \\
\text{Since } s > 0
\end{align*}
\]

**Ex 5 (#34)** Failing Distance

a) A ring is dropped from a helicopter at an altitude of 75 m. Approximately how long does it take the ring to reach the ground? \( 3.9 \text{ sec} \)

\[
\begin{align*}
4.9t^2 + 0 &= 75 \\
4.9t^2 &= 75 \\
t &= \frac{\sqrt{75}}{4.9} \\
\Rightarrow t &= \pm \frac{\sqrt{75}}{4.9} \\
&= \pm 3.9 \text{ sec}
\end{align*}
\]

b) A coin is tossed downwards with an initial velocity of 30 m/sec from an altitude of 75 m. Approximately how long does it take the coin to reach the ground? \( 1.9 \text{ sec} \)

\[
\begin{align*}
4.9t^2 + 30t &= 75 \\
4.9t^2 + 31t - 75 &= 0 \\
a &= 4.9, \ b = 30, \ c = -75
\end{align*}
\]

\[
\begin{align*}
t &= \frac{-30 \pm \sqrt{900 - 4(4.9)(-75)}}{2(4.9)} \\
&= \frac{-30 \pm \sqrt{2870}}{9.8} \\
&= \frac{-30 \pm 53.7}{9.8} \\
&= 1.9 \text{ sec}
\end{align*}
\]

c) Approximately how far will an object fall in 2 sec, if thrown downward at an initial velocity of 20 m/sec from a helicopter. \( 59.6 \text{ m} \)

\[
\begin{align*}
4.9t^2 + 20t &= 0 \\
t &= 2 \\
4.9(2)^2 + 20(2) &= 8 \\
19.6 + 40 &= 59.6 \text{ m}
\end{align*}
\]

Omit #43 from (textbook) HW
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Solve and check answers.

1) \( x^4 - 17x^2 + 16 = 0 \)
   
   Let \( u = x^2 \)

   \( u^2 - 17u + 16 = 0 \)

   \( (u-16)(u-1) = 0 \)

   \( u = 16 \quad \text{or} \quad u = 1 \)

   \( x^2 = 16, \quad x^2 = 1 \)

2) \( r - 2\sqrt{r} - 6 = 0 \)

   Let \( u = \sqrt{r} \)

   \( u^2 - 2u - 6 = 0 \)

   \( u = \frac{2 \pm \sqrt{4+24}}{2} = 2 \pm 2\sqrt{5} \)

   \( u = 2 + 2\sqrt{5} \quad \text{or} \quad u = 2 - 2\sqrt{5} \)

   \( r = (2 + 2\sqrt{5})^2 \quad r = (2 - 2\sqrt{5})^2 \)

3) \( (x^2 - 2)^2 - 12(x^2 - 2) + 20 = 0 \)

   Let \( u = x^2 - 2 \)

   \( u^2 - 12u + 20 = 0 \)

   \( (u-2)(u-10) = 0 \)

   \( u = 2 \quad \text{or} \quad u = 10 \)

   \( x^2 = 2, \quad x^2 = 12 \)

   \( x = \pm \sqrt{2}, \quad x = \pm \sqrt{12} \)

4) \( 2x^2 - x - 1 = 0 \)

   Let \( u = x^2 \)

   \( u^2 - u - 1 = 0 \)

   \( (2u+1)(u-1) = 0 \)

   \( u = -\frac{1}{2}, \quad u = 1 \)

   \( x = -\frac{1}{2}, \quad x = 1 \)

5) \( w^{2/3} - 2w^{1/3} - 8 = 0 \)

   Let \( x = w^{1/3} \)

   \( x^2 - 2x - 8 = 0 \)

   \( (x-4)(x+2) = 0 \)

   \( x = 4, \quad x = -2 \)

   \( w = 4^3, \quad w = -2^3 \)

   \( w = 64, \quad w = -8 \)

Ex. 7 Find all x-intercepts of each function.

* \( f(x) = x^{1/2} - x^{1/4} - 6 \)

Set \( y = f(x) = 0 \) and solve for \( x^{1/4} \)

\( 0 = x^{1/2} - x^{1/4} - 6 \)

\( (u-3)(u+2) = 0 \)

\( u = 3 \quad \text{or} \quad u = -2 \)

\( x^{1/4} = 3, \quad x^{1/4} = -2 \)

\( x = 81 \)

Check: \( (81)^{1/2} - (81)^{1/4} = 0 \)

*Prac Prob*

\( g(x) = (3 + \sqrt{x})^2 + 3(3 + \sqrt{x}) - 10 \)

Let \( u = (3 + \sqrt{x}) \)

\( y = u^2 + 3u - 10 \)

\( x = \text{int} \)

\( y = 0 \)

\( u = -5, \quad u = 2 \)

\( 3 + \sqrt{x} = -5, \quad 8 + \sqrt{x} = 2 \)

\( \sqrt{x} = -2, \quad \sqrt{x} = 1 \)

\( x = \text{not possible} \)

No y-ints
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**Quadratic Functions and Their Graphs**

Vertex Form: \( f(x) = a(x - h)^2 + k \)

The graph of \( f(x) = a(x - h)^2 + k \) has the same shape as the graph of \( y = ax^2 \).

- If \( h > 0 \), the graph of \( y = ax^2 \) is shifted \( h \) units to the right. \( \text{ex} \ f(x) = 2(x - 3)^2 \)
- If \( h < 0 \), the graph of \( y = ax^2 \) is shifted \( |h| \) units to the left. \( \text{ex} \ f(x) = 2(x - (-3))^2 = 2(x + 3)^2 \)
- If \( k > 0 \), the graph of \( y = ax^2 \) is shifted \( k \) units up. \( \text{ex} \ f(x) = 2x^2 + 3 \)
- If \( k < 0 \), the graph of \( y = ax^2 \) is shifted \( |k| \) units down. \( \text{ex} \ f(x) = 2x^2 + (-3) = 2x^2 - 3 \)
- The vertex is \((h, k)\) and the axis of symmetry is \( x = h \).
- If \( a > 0 \), the parabola opens upward and the minimum function value is \( k \).
- If \( a < 0 \), the parabola opens downward and the maximum function value is \( k \).

**Ex 1** Graph.

a) \( f(x) = -3x^2 \) \( \text{Vertex} \ (0,0) \)

b) \( g(x) = \frac{1}{4}x^2 \) \( \text{Vertex} \ (0,0) \)

**Ex 2** For each of the following, graph the function, label the vertex, draw the axis of symmetry, and the maximum/minimum value (extremum).

a) \( g(x) = 3(x - 5)^2 \) \( \text{vertex} \ (5,0) \) \( \text{min} \)

b) \( h(x) = -(x - 1)^2 + 2 \) \( \text{vertex} \ (1,2) \) \( \text{max} \)
c) \( r(x) = \frac{3}{2}(x + 2)^2 - 4 \)
\[ V(-2, -4) \]

Ex 3 Without graphing, find the vertex, the axis of symmetry, and the extremum.

- **a)** \( f(x) = 2(x - 1)^2 - 10 \)
  - Vertex: \((1, -10)\)
  - Axis of Symm: \(x = 1\)
  - Minimum: \(-10\)

- **b)** \( f(x) = 2(x - 0.01)^2 + \sqrt{15} \)
  - Vertex: \((0.01, \sqrt{15})\)
  - Axis of Symm: \(x = 0.01\)
  - Maximum: \(\sqrt{15}\)

Ex 4 Write an equation for a function having a graph with the same shape as the graph of \( f(x) = \frac{3}{5}x^2 \), but with the given point as the vertex.

- **a)** \((9, -6)\)
  - \( f(x) = a(x - h)^2 + k \)
  - \( a = \frac{3}{5} \)
  - \( h = 9 \)
  - \( k = -6 \)

- **b)** \((-1.5, \frac{3}{5})\)
  - \( f(x) = a(x - h)^2 + k \)
  - \( a = \frac{3}{5} \)
  - \( h = -1.5 \)
  - \( k = \frac{3}{5} \)

Ex 5 Write the equation of the parabola that has the shape of \( f(x) = 2x^2 \) or \( g(x) = -2x^2 \) and has a maximum/minimum value at the specified point.

- Minimum; \((-4, 0)\)
  - \( f(x) = a(x - h)^2 + k \)
  - \( h = -4 \)
  - \( k = 0 \)
  - \( a = 2 \)

So \( f(x) = 2(x + 4)^2 \)
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More About Graphing Quadratic Functions

The Vertex of a Parabola

The vertex of the parabola given by \( f(x) = ax^2 + bx + c \) is \( \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \) or \( \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) \).

- The x-coordinate of the vertex is \(-\frac{b}{2a}\).
- The axis of symmetry is \( x = -\frac{b}{2a} \).
- The second coordinate of the vertex is most commonly found by computing \( f\left(-\frac{b}{2a}\right) \).

Ex 1: Complete the square to write each function in the form \( f(x) = a(x-h)^2 + k \).

a) \( f(x) = x^2 + 5x + 3 \)

\[ f(x) = \left(x + \frac{5}{2}\right)^2 - \frac{1}{4} \]

b) \( f(x) = 2x^2 - 5x + 10 \)

\[ f(x) = 2\left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) + 10 - \frac{25}{8} \]

Ex 2: For each quadratic function, (a) find the vertex and axis of symmetry, (b) find the maximum/minimum function value, and (c) graph the function.

a) \( f(x) = x^2 - 10x + 21 = \frac{1}{2}(x-5)^2 - 4 \)

b) \( f(x) = -3x^2 - 7x + 2 \)

Ex 3: Find the x- and y-intercepts of \( f(x) = x^2 - 10x + 21 \).

- y-intercept: \( y = 0 \Rightarrow x = 10 \) or \( x = 21 \)
- x-intercept: \( y = 0 \Rightarrow 0 = x^2 - 10x + 21 \)

Vertex:
\[ (-\frac{b}{2a}, f(-\frac{b}{a})) = (5, -4) \]

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