Math 120 Intermediate Algebra

Absolute Value Equations and Inequalities

Defn The absolute value of $x$, denoted $|x|$, is defined as $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

The Absolute Value Principle for Equations
Let $p$ be a positive number and let $X$ be an algebraic expression.

- **a)** The solutions of $|X| = p$ are those that satisfy $X = -p$ or $X = p$.
- **b)** The equation $|X| = 0$ is equivalent to $X = 0$.
- **c)** The equation $|X| = -p$ has no solution.

Principles for Solving Absolute Value Problems
Let $p$ be a positive number and let $X$ be an algebraic expression.

- **a)** The solutions of $|X| = p$ are those that satisfy $X = -p$ or $X = p$.
- **b)** The solutions of $|X| < p$ are those numbers that satisfy $-p < X < p$.
- **c)** The solutions of $|X| > p$ are those numbers that satisfy $X > p$ or $X < -p$.
- **d)** The solutions of $|X| = |Y|$ are those numbers that satisfy $X = Y$ and $X = -(Y)$.

Ex 1 Solve.

a) $|5x + 2| = 7$  

b) $|5x| + 2 = 7$

Ex 2 Let $f(x) = \frac{1-2x}{5}$. Find all $x$ for which $f(x) = 2$.

Ex 3 Solve.

a) $|5t + 7| = |4t + 3|$  

b) $|6 - 5t| = |5t + 8|$

Ex 4 Solve and graph.

a) $1 - \frac{1}{2} |2a + 5| \geq -3$  

b) $\left| \frac{2-5x}{4} \right| \geq \frac{2}{3}$