Math 401 Practice Final

1. Find the area bounded by \( x = 3 - y^2 \) and \( x = y + 1 \).

2. Find the volume of the solid formed when the region bounded by \( y = \sin x \), \( y = 0 \), \( 0 \leq x \leq \pi \) is revolved about the \( x \)-axis.

3. Find the volume of the solid formed when the region is bounded by \( y = 2 - x^2 \) and \( y = 1 \) is revolved about the line \( y = 1 \).

4. Find the volume of the solid formed when the region bounded by \( y = x^2 + 4 \), \( y = 8 \), \( x = 0 \) is revolved about the \( y \)-axis.

5. A solid has as its base in the \( x-y \) plane the region bounded by \( x^2 + y^2 = 16 \). Find the volume of the solid formed if every cross section perpendicular to the \( y \)-axis is a semicircle.

6. A right circular cylindrical tank is full of water. It has a radius of 8' and a height of 8'. Find the work required to pump all of the water over the top (water weighs 62.4 lbs/ft\(^3\)).

7. Integrate.
   
   (a) \[ \int x \cos x \, dx \]  
   (b) \[ \int \tan^{-1} x \, dx \]  
   (c) \[ \int \sin^5 x \cos^2 x \, dx \]  
   (d) \[ \int \tan^3 x \sec^3 x \, dx \]  
   (e) \[ \int \frac{x}{\sqrt{x^2 + 9}} \, dx \]  
   (f) \[ \int \frac{1}{\sqrt{25 - x^2}} \, dx \]  
   (g) \[ \int \frac{x + 2}{x^2 - 4x} \, dx \]  
   (h) \[ \int \frac{3}{x^2 + x - 2} \, dx \]  
   (i) \[ \int_1^\infty \frac{\ln x}{x} \, dx \]  

8. Find limits.
   
   (a) \[ \lim_{x \to 1} \frac{\ln x^2}{x + 1} \]  
   (b) \[ \lim_{x \to 0} \frac{\sin 2x}{\sin 3x} \]  
   (c) \[ \lim_{x \to \infty} \frac{x^2}{e^x} \]  

9. Determine if the series converges (absolutely or conditionally) or diverges.

   (a) \[ \sum_{n=1}^{\infty} \frac{n}{n + 1} \]  
   (b) \[ \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n \]  
   (c) \[ \sum_{n=1}^{\infty} \frac{1}{n \sqrt{n}} \]  
   (d) \[ \sum_{n=1}^{\infty} \frac{\ln n}{n} \]  
   (e) \[ \sum_{n=1}^{\infty} \frac{n}{n^2 - 1} \]  
   (f) \[ \sum_{n=1}^{\infty} \frac{1}{n(n^2 + 1)} \]  
   (g) \[ \sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{n!}\right) \]  
   (h) \[ \sum_{n=1}^{\infty} (-1)^n \frac{1}{n \sqrt{n}} \]  
   (i) \[ \sum_{n=1}^{\infty} \frac{(-1)^n (7)^n}{n!} \]  
   (j) \[ \sum_{n=1}^{\infty} \frac{n!}{n3^n} \]  
   (k) \[ \sum_{n=1}^{\infty} (-1)^n \frac{1}{n!} \]  

10. Write a Maclaurin Series for \( f(x) = \sin x \).
11. Write \(16x^2 + 25y^2 - 64x + 150y + 279 = 0\) in standard form and graph.

12. Sketch \(x = \sqrt{t}, \ y = t - 2\).

13. Find the slope and concavity at \(t = 1\) for \(x = t + 1, \ y = t^2 + 3t\).

14. Find the arc length for \(x = t^2, \ y = 2t, \ 0 \leq t \leq 2\).

15. Find the surface area for \(x = t, \ y = 4 - 2t, \ 0 \leq t \leq 2\) revolved about the \(y\)-axis.

16. Graph.

\[
\begin{align*}
(a) \quad r &= \sin 3\theta \\
(b) \quad r &= 2 + 3\cos \theta \\
(c) \quad r &= 3 - 3\cos \theta
\end{align*}
\]

17. Find the slope of \(r = \sin 2\theta\) at \(\theta = \frac{\pi}{6}\).

18. Find the length of \(r = 3 + 3\cos \theta\) from \(\theta = 0\) to \(\pi\).

19. Find the area of one petal of \(r = \sin 2\theta\).

20. Find the surface area when \(r = 2\cos \theta\) is revolved about \(\theta = \frac{\pi}{2}\).

21. Identify, convert to rectangular coordinates and graph \(r = \frac{1}{1 - 2\sin \theta}\).