Math 370 Precalculus

Sec 10.1: Conics

We will study all 4 types of conic sections, which are curves that result from the intersection of a right circular cone and a plane that does not contain the vertex. (If the plane contains the vertex, the intersection of the plane and the cone is a point, line, or a pair of intersection lines. These are usually called degenerate conics.)

Sec 1.4: Circles

Defn A circle is a set of points in the xy-plane that is a fixed distance \( r \) from a fixed point \((h, k)\). The fixed distance \( r \) is called the radius, and the fixed point \((h, k)\) is called the center.

**Standard Equation of a Circle**

\[
(x - h)^2 + (y - k)^2 = r^2
\]

Note: The equation \(x^2 + y^2 + ax + by + c = 0\) is that of a circle, a point, or has no graph at all. If it’s the graph of a circle, it is referred to as the **general form of the equation of a circle**.

Ex 1 (#8) Find the center and radius of the circle. Write the standard form of the equation.

Ex 2 (#30) (a) Find the center, \((h, k)\), and the radius, \(r\). (b) Graph the circle. (c) Find the intercepts, if any. \[ x^2 + y^2 + x + y - \frac{1}{2} = 0 \]
Ex 3  (38) Find the standard form of the equation of the circle.
Center (−3, 1) and tangent to the y-axis

Sec 10.3: The Ellipse

**Defn**  An **ellipse** is the collection of all points in the plane, the sum of whose distances from two fixed points, called the **foci**, is a constant. Note: The constant is 2a.

The line containing the foci is called the **major axis**. The midpoint of the line segment joining the foci is the **center** of the ellipse. The line through the center and perpendicular to the major axis is the **minor axis**. The **vertices** are the two points of intersection of the ellipse and major axis.

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**Equation of Ellipse with Center at (h, k); Major Axis Along x-Axis**

\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \ a > b > 0
\]

**Vertices:** \((h \pm a, k)\)  
**Foci:** \((h \pm c, k)\)

**Equation of Ellipse with Center at (h, k); Major Axis Along y-Axis**

\[
\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \ a > b > 0
\]

**Vertices:** \((h, k \pm a)\)  
**Foci:** \((h, k \pm c)\)

Ex 4  (24) Find the vertices and foci of the ellipse. Graph the equation.  

\[4y^2 + 9x^2 = 36\]
Ex 5 (27) Find an equation for each ellipse. Graph each equation.
Center at (0,0); focus at (3,0); vertex at (5,0)

Ex 6 (40) Write an equation for the ellipse.

Ex 7 (50) Analyze the equation; that is, find the center, foci, and vertices of the ellipse. Graph the equation.

\[4x^2 + 3y^2 + 8x - 6y = 5\]
Ex 8 (#72) If a source of light or sound is placed at one focus of an ellipse, the waves transmitted by the source will reflect off the ellipse and concentrate at the other focus. This is the principle behind *whispering galleries*, which are rooms designed with elliptical ceilings. Jim, standing at one focus of a whispering gallery, is 6 feet from the nearest wall. His friend is standing at the other focus, 100 feet away. What is the length of this whispering gallery? How high is its elliptical ceiling at the center? **112 ft; 25.2 ft**

Sec 10.4: The Hyperbola

Defn. A **hyperbola** is the collection of points in the plane, the difference of whose distances from two fixed points, called the **foci**, is a constant. Note: The constant is 2a.

The line containing the foci is called the **transverse axis**. The midpoint of the line segment joining the foci is the **center** of the hyperbola. The line through the center and perpendicular to the transverse axis is the **conjugate axis**. The hyperbola consists of two separate curves, called **branches**.

**Equation of Hyperbola with Center at** \((h, k)\);

**Transverse Axis Parallel to the** \(x\)-**Axis**

\[
\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1
\]

\(c^2 = a^2 + b^2 \quad c > a\)

**Vertices:** \((h \pm a, k)\)

**Foci:** \((h \pm c, k)\)

**Asymptotes:** \(y - k = \pm \frac{b}{a} (x - h)\)

**Equation of Hyperbola with Center at** \((h, k)\);

**Transverse Axis Parallel to the** \(y\)-**Axis**

\[
\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1
\]

\(c^2 = a^2 + b^2 \quad c > a\)

**Vertices:** \((h, k \pm a)\)

**Foci:** \((h, k \pm c)\)

**Asymptotes:** \(y - k = \pm \frac{a}{b} (x - h)\)

Ex 9  Find an equation for the hyperbola described. Graph the equation.

(420) Center at \((0, 0)\); focus at \((-3, 0)\); vertex at \((2, 0)\)
(23) Vertices at \((0, \pm 6)\); asymptote the line \(y = 2x\)

**Ex 10** (#36) Write an equation for the hyperbola.

**Ex 11** (#47) Find the center, transverse axis, vertices, foci, and asymptotes. Graph the equation.

\[
\frac{(x - 2)^2}{4} - \frac{(y + 3)^2}{9} = 1
\]

(#49)

\[(y - 2)^2 - 4(x + 2)^2 = 4\]
Ex 12  Graph
\[ \frac{(x + 2)^2}{9} + \frac{(y - 3)^2}{25} = 1 \]

Ex 13  Write an equation of the ellipse with foci at \((1, -4)\) and \((5, -4)\) and with one vertex at \((8, -4)\).
Sec 10.2: The Parabola

Defn A parabola is a collection of points $P$ in the plane that are the same distance from a fixed point $F$ as they are from a fixed line $D$. The point $F$ is called the focus of the parabola, and the line $D$ is called the directrix. As a result, a parabola is the set of points $P$ for which $d(P, F) = d(P, D)$.

The line through the focus and perpendicular to the directrix is called the axis of symmetry. The point of intersection of the parabola with its axis of symmetry is called the vertex $V$. The line segment joining the points on the parabola "above/below" or to the "left/right" of the focus is called the latus rectum.

Use the distance formula to derive the equation of a parabola.
**Equation of Parabola with Vertex** $(h, k)$;

**Opens Left/Right**

$$(y - k)^2 = \pm 4a(x - h)$$

or

$$x = \pm \frac{1}{4a}(y - k)^2 + h$$

Focus: $(h \pm a, k)$

Directrix: $x = h \pm a$

**Equation of Parabola with Vertex** $(h, k)$;

**Opens Up/Down**

$$(x - h)^2 = \pm 4a(y - k)$$

or

$$y = \pm \frac{1}{4a}(x - h)^2 + k$$

Focus: $(h, k \pm a)$

Directrix: $y = k \pm a$

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**Ex 14** Find the equation of the parabola described. Find the two points that define the latus rectum, and graph the equation.

**#22** Focus at $(-4,0)$; vertex at $(0,0)$

**#30** Vertex at $(4,-2)$; focus at $(6,-2)$
(36) Focus at \((-4,4)\); directrix the line \(y = -2\)

**Ex 15** Find the vertex, focus, and directrix of each parabola. Graph the equation.

(39) \(y^2 = -16x\)

(45) \((y + 3)^2 = 8(x - 2)\)

(50) \(y^2 - 2y = 8x - 1\)
Ex 16 (Prac Prob) A cable TV receiving dish is in the shape of a paraboloid of revolution. Find the location of the receiver, which is placed at the focus, if the dish is 6 feet across at its opening and 2 feet deep. 1.125 ft or 13.5 in from the base of dish along axis of symm

Sec 11.6: Systems of Nonlinear Equations

Ex 17 Graph each equation of the system. Then solve the system to find the points of intersection.

\[
\begin{align*}
\text{(#8)} & \quad y = \sqrt{4 - x^2} \\
\text{(#15)} & \quad y = 3x - 5 \\
\text{(#20) Prac Prob} & \quad \begin{cases} 
  x^2 + y^2 = 5 \\
  x^2 = y \\
  xy = 1 \\
  (1,1)
\end{cases}
\end{align*}
\]

Ex 18 Solve each system. Use any method you wish.

\[
\begin{align*}
\text{(#19)} & \quad \begin{cases} 
  xy = 4 \\
  x^2 + y^2 = 8
\end{cases}
\end{align*}
\]
Ex 19 (#86) Prac Prob  

A rectangular piece of cardboard, whose area is 216 square centimeters, is made into a cylindrical tube by joining together two sides of the rectangle. See the figure. If the tube is to have a volume of 224 cubic centimeters, what size cardboard should you start with? 16.57 cm by 13.03 cm
Sec 11.7: Systems of Inequalities

Ex 20  Graph each inequality.
\[ x^2 + y^2 < 4 \quad y > x^2 + 4 \quad (\#22) \quad xy \leq 1 \]

Ex 21  (\#31)  Graph the system of linear inequalities.
\[ \begin{cases} 2x + y \geq -2 \\ 2x + y \geq 2 \end{cases} \]

Ex 22  (\#36)  Graph each system of inequalities.
\[ \begin{cases} x^2 + y^2 \geq 9 \\ x + y \leq 3 \end{cases} \]
\begin{align*}
\text{(39)} & \begin{cases} 
    x^2 + y^2 \leq 16 \\
    y \geq x^2 - 4
\end{cases} \\
\text{(42)} & \begin{cases} 
    y + x^2 \leq 1 \\
    y \geq x^2 - 1
\end{cases}
\end{align*}
Sec 10.7: Plane Curves and Parametric Equations

All the above are examples of equations that are not ______ because they don't pass the ______. Each is a collection of points \((x, y)\) in the xy-plane and are called plane curves.

We can find an equation to describe the circle or parabola but cannot do so using a single equation (by solving for \(y\)).

So...

We use parametric equations to re-represent plane curves. This new representation provides more information about the graph, namely the orientation.

Defn

Let \(x = f(t)\) and \(y = g(t)\), where \(f\) and \(g\) are functions whose common domain is some interval \(I\). The collection of points defined by

\[
(x, y) = (f(t), g(t))
\]

is called a plane curve. The equations

\[
\begin{align*}
x &= f(t) \\
y &= g(t)
\end{align*}
\]

are called parametric equations of the curve. The variable \(t\) is called a parameter.

Ex 23 Refer to Parametric Equations Handout (Day 1: 1, 3, 4, 6, 8; Day 2: 9, 10)

Ex 24 Find parametric equations that define the curve shown. Are there multiple answers? Provide several more.
Ex 25 (#50) Alice throws a ball straight up with an initial speed of 40 feet per second from a height of 5 feet.

a) Find parametric equations that describe the motion of the ball as a function of time.

b) How long is the ball in the air?

c) When is the ball at its maximum height? Determine the maximum height of the ball.

d) Simulate the motion of the ball by graphing the equations found in part (a).

Use $x = 3; \ y = -16t^2 + 40t + 5; \ Tstep = 0.01; \ 0 \leq t \leq 2.7$
Ex 26 (Prac Prob) Jodi’s bus leaves at 5:30 PM and accelerates at the rate of 3 meters per second per second. Jodi, who can run 5 meters per second, arrives at the bus station 2 seconds after the bus has left and runs for the bus.

a) Find parametric equations that describe the motions of the bus and Jodi as a function of time. 
[Hint: The position $s$ at time $t$ of an object having acceleration $a$ is $s = \frac{1}{2}at^2$.

b) Determine algebraically whether Jodi will catch the bus. If so, when?

c) Simulate the motion of the bus and Jodi by simultaneously graphing the equations found in part a).

Ex 27 Find a rectangular equation for the plane curve defined by the parametric equations. Be sure to indicate the restrictions on $x$.

a) $x = 2t, \ y = t + 3; \ -2 \leq t \leq 3$

b) $x = t^3 + 1, \ y = t^3 - 20; \ -2 \leq t \leq 2$

c) $x = 3 \tan t, \ y = 4 \sec t; \ 0 \leq t \leq 2\pi$