Sec 4.1: Inequalities and Applications

Defns - recall
- An inequality is any sentence containing <, >, ≤, ≥, or ≠.
- Any value for the variable that makes an inequality true is a solution.
- The set of all solutions is called the solution set.

Interval and Set-Builder Notation – recall

\[
(a, b) = \{x | a < x < b\} \quad \quad [a, b] = \{x | a \leq x \leq b\}
\]

\[
(a, b] = \{x | a < x \leq b\} \quad \quad [a, b) = \{x | a \leq x < b\}
\]

\[
(a, \infty) = \{x | x > a\} \quad \quad (-\infty, a) = \{x | x < a\}
\]

**Multiplication Principle for Inequalities**
For any real numbers \(a\) and \(b\), and for any negative number \(c\),

\[a < b\] is equivalent to \(ac > bc\);
\[a > b\] is equivalent to \(ac < bc\).

Similar statements hold for \(<\) and \(\geq\).

**Ex 1** Solve and graph. Write the solution set using both set-builder notation and interval notation.

a) \(\frac{5}{6}x \geq -2\) 

b) \(\frac{2t-9}{-3} > 0\)

**Ex 2** Solve and write solutions using both types of notation.

a) \(8x - 3(3x + 2) - 5 \geq 3(x + 4) -\)

b) \(\frac{1}{3}(6x + 24) - 20 > -\frac{1}{4}(12x - )\)

**Ex 3** Find the domain of \(f(x) = \sqrt{8 -}\).
Sec 4.2: Intersections, Unions, and Compound Inequalities

Defns
- Two inequalities joined by the word “and” or “or” are called compound inequalities.
- The intersection of two sets $A$ and $B$ is the set of all elements that are common to both $A$ and $B$, and is denoted $A \cap B$.
- The union of two sets $A$ and $B$ is the collection of elements belonging to $A$ or $B$ (or both), and is denoted $A \cup B$.
- When two are more sentences are joined by the word and to make a compound sentence, the new sentence is called a conjunction of the sentences.
- When two are more sentences are joined by the word or to make a compound sentence, the new sentence is called a disjunction of the sentences.

Ex 1 Find the intersection $A \cap B$ and the union $A \cup B$ where $A = \{ \}$ and $B = \{ \}$.

Ex 2 Graph and write interval notation for each compound inequality.
   a) $x \leq -5 \text{ or } x >$  
   b) $x \geq -3 \text{ and } x <$  
   c) $x \geq 0 \text{ and } x \leq$

Ex 3 Solve and graph each solution set.
   a) $-3 \leq \frac{1}{2}(x - 2) \leq$  
   b) $-10 \leq f(x) \leq -8$, where $f(x) = \frac{x+6}{2}$

   c) $5 - 3a \leq 8 \text{ or } 2a + 1 >$
Ex 4 Use interval notation to write the domain of \( f(x) = \frac{x-1}{3x+1} \)

Sec 4.3: Absolute Value Equations and Inequalities

Defn The **absolute value** of \( x \), denoted \( |x| \), is defined as

\[
|x| = \begin{cases} 
  x, & x \geq 0 \\
  -x, & x < 0 
\end{cases}
\]

The Absolute Value Principle for Equations

Let \( p \) be a positive number and let \( X \) be an algebraic expression.

a) The solutions of \( |X| = p \) are those that satisfy \( X = -p \) or \( X = p \).

b) The equation \( |X| = 0 \) is equivalent to \( X = 0 \).

c) The equation \( |X| = -p \) has no solution.

Principles for Solving Absolute Value Problems

Let \( p \) be a positive number and let \( X \) be an algebraic expression.

a) The solutions of \( |X| = p \) are those that satisfy \( X = -p \) or \( X = p \).

b) The solutions of \( |X| < p \) are those numbers that satisfy \( -p < X < p \).

c) The solutions of \( |X| > p \) are those numbers that satisfy \( X > p \) or \( X < -p \).

d) The solutions of \( |X| = |Y| \) are those numbers that satisfy \( X = Y \) and \( X = -Y \).

Ex 1 Solve.

a) \( |5x + 2| = \)  

b) \( |5x| + 2 = \)

Ex 2 Let \( f(x) = \left| \frac{1-2x}{5} \right| \). Find all \( x \) for which \( f(x) = \) .

Ex 3 Solve.

a) \( |5t + 7| = |4t + | \)  

b) \( |6 - 5t| = |5t + | \)
Ex 4 Solve and graph.

a) \( 1 - \frac{1}{2} |2a + 5| \geq - \)

b) \( \left| \frac{2-5x}{4} \right| \geq \frac{2}{3} \)

Sec 4.4: Inequalities in Two Variables

Recall – A linear equation has the form \( Ax + By = C \). When = is replaced by <, >, \( \leq \), or \( \geq \), a linear inequality is formed.

The graph of a linear equation is a line. The graph of a linear inequality is a half-plane with a boundary that is a line.

Ex 1 Graph on a plane. (Graph the region.)

a) \( 3x + 4y \leq \)

b) \( 3x - 2 < 5x + \)

c) \( 0 \leq x < \)

Ex 2 Graph each system.

a) \( \begin{cases} y \geq \\ y \leq -x + \end{cases} \)

b) \( \begin{cases} 2y - x \leq \\ y - 3x \geq - \\ y \geq - \end{cases} \) Find the coordinates of the vertices formed.