Math 120 Intermediate Algebra

Sec 2.1: Graphs

Review:
- axes – x/y, first/second
- origin
- ordered pairs
- coordinates – x/y, first/second
- Cartesian Coordinate System
- quadrants

Ex 1  Determine whether the ordered pair is a solution of the equation.  
(−2,17); x⁴ + y = 1

Ex 2  Graph.

a)  
\[ y = \frac{-3}{4} x + 2 \]

b)  
\[ y = x^2 \]

c)  
\[ y = x^2 + 2 \]

d)  
\[ y = |x| \]

e)  
\[ y = |x| + 2 \]

f)  
If \( y = x^3 \) looks like snake/disco, make conjecture about what \( y = x^3 + 2 \) looks like.

Sec 2.2: Functions

Defns
- A relation is a set of ordered pairs.
- A function is a relation where each first coordinate corresponds to exactly one second coordinate.

Informally, a function is a special kind of correspondence between two sets.

Examples:
To each person there corresponds a date of birth.
To each number there corresponds double that number.

First set – domain; Second set – range.

Defns
- The domain of a relation (or function) is the set of all first coordinates.
- The range of a relation (or function) is the set of all second coordinates.

Example: Domain: Girls, Range: Boys, Correspondence: Dating

Ex 1  Is the correspondence a function?
Ex 2  Write the domain, range, and determine if the correspondence is a function.
{(0,7), (4,8), (7,0), (8,4), (2,4)}

Function Introduction

Function Notation: \( f(x) \) is read “\( f \) of \( x \)”
or “\( f \) at \( x \)”
or “the value of \( f \) at \( x \)”

Function Machine

Think of \( f(x) \) at the \( y \)-value or output. (\( x \) is the input). Consider \( f(x) = 2x \) using function machine analogy and \( f(2) = 4 \).

**Warning!** \( f(x) \) does not mean \( f \) times \( x \). In the above example, \( f(2) \) does not mean \( f \) times 2.

Ex 3  Given the following functions, find the indicated values.

\[
\begin{align*}
\text{Ex 3} & \quad \text{Given the following functions, find the indicated values.} \\
& \\
g(-1) & \quad h\left(-\frac{1}{3}\right) \\
r(-4) & \quad r(2) \\
p\left(\frac{15}{2}\right) & \quad p(-1) \\
& \quad g(x + h) \\
f(3 + h) & \quad f(h) - 3 \\
t(7) & \quad t(a + b)
\end{align*}
\]
Let $a, b \in \mathbb{R}$ or let $a$ or $b$ represent $\pm \infty$.

### Interval Notation vs. Set-Builder Notation

<table>
<thead>
<tr>
<th>Interval Notation</th>
<th>Set-Builder Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(a, b)$</td>
<td>${x</td>
</tr>
<tr>
<td>$[a, b]$</td>
<td>${x</td>
</tr>
<tr>
<td>$(a, b]$</td>
<td>${x</td>
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<tr>
<td>$[a, b)$</td>
<td>${x</td>
</tr>
<tr>
<td>$(a, \infty)$</td>
<td>${x</td>
</tr>
<tr>
<td>$(-\infty, a)$</td>
<td>${x</td>
</tr>
</tbody>
</table>

*Recommended but do not require this notation until ch4*

### Bad Notation

- $(\ , \infty]$  
- $[-\infty, \ )$  
- $(\ , \infty)$  
- $(\ , -\infty)$  
- $[\infty, \ )$  
- $(\ , -\infty)$  

- $(-4, -11)$  
- $(3, 0)$

### Ex 4

For each graph, determine (a) $f(1)$; (b) the domain; (c) any $x$-values for which $f(x) = 2$; and (d) the range. Use set-builder and interval notation.

- a) Page 92 #28
- b) Page 92 #30

### Ex 5

Determine the domain and range of each function. Use set-builder and interval notation.

- a) Page 92 #40
- b) Page 92 #42
- c) Page 92 #44
- d) Page 93 #48
The Vertical Line Test
If it is possible for a vertical line to cross a graph more than once, then the graph is not the graph of a function.

Ex 6 Determine whether each of the following is the graph of a function.

a) Page 93 #50

b) Page 93 #52

c) Page 93 #54

Finding the Domain of a Function
Denominator cannot be zero. Will see more examples later.

Ex 7 Find the domain of each function. Use set-builder and interval notation.

a) \( f(x) = \frac{11}{x - 4} \)  

b) \( g(x) = x^2 - 7 \)  

c) \( f(x) = \frac{x + 3}{2x + 1} \)  

d) \( h(x) = |3x - 4| \)  

e) \( r(x) = \frac{3x + 1}{2} \)  

f) \( l(x) = \frac{x + 1}{x^2 - 9} \)  

g) \( s(x) = \frac{x + 1}{x^2} \)  

h) \( a(x) = \frac{x + 1}{x^2 + 1} \)  

i) \( f(x) = \frac{7}{6 - x} \)  

j) \( b(x) = 2x + 1 \)  

k) \( c(x) = |3 - 4x| \)  

l) \( d(x) = \frac{1}{2}x - \frac{2}{x} \)
Ex 8  Find each.
\[ f(2) = \quad f(0) = \quad f(-4) = \quad f(3) = \quad f(-1) = \]

Ex 9  Find the domain or range (or both).

Domain:__________
Range:__________

Domain:__________
Range:__________

Domain:__________
or__________
Range:__________
or__________

Ex 10  Find each.
\[ f(x) = -3 \quad f(x) = 4 \quad f(x) = -2 \]
Sec 2.3: Linear Functions: Slope, Graphs, and Models

**Defn** The slope of the line passing through \((x_1, y_1)\) and \((x_2, y_2)\) is given by

\[ m = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{the diff in } y}{\text{the diff in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2} \]

- positive slope
- negative slope
- zero slope
- no slope/undefined slope

**Ex 1** Graph. \(f(x) = -\frac{1}{2}x - 5\)

**Ex 2** Find the slope of the line containing \((-\frac{5}{2}, -\frac{11}{3})\) and \((-\frac{8}{3}, -\frac{21}{2})\).

**Ex 3** Find a linear function whose graph has slope \(-\frac{3}{4}\) and \(y\)-intercept \((0, -5)\).

**Ex 4** Write an equation of each line described. Express answers in slope-intercept form if possible.

a) The line parallel to the \(y\)-axis passing through \((\sqrt{2}, \frac{\pi}{2})\)

b) The line perpendicular to the \(y\)-axis passing through \((\sqrt{2}, \frac{\pi}{2})\)

c) The line passing through the points \((-3, 9)\) and \((-4, 2)\).

d) The line passing through \((8, 0)\) and parallel to the line \(2x - 3y = -6\)

e) The line passing through \((8, 0)\) and perpendicular to the line \(2x - 3y = -6\)

**Ex 5** *If time permits* Page 105 #62
For each graph, find the rate of change. Remember to use appropriate units.

**Ex 6** *If time permits* Given that \(f(x) = mx + b\), is \(f(cd) = f(c)f(d)\)?

Sec 2.4: Another Look At Linear Graphs

**Horizontal Lines**
- The slope of a horizontal line is _________.
- The graph of any function of the form \(f(x) = b\) or \(y = b\) is a _________ line that crosses the \(y\)-axis at (__, __).

**Vertical Lines**
- The slope of a vertical line is _________.
- The graph of any equation of the form \(x = a\) is a _________ line that crosses the \(x\)-axis at (__, __).

**Ex 1** Find the slope of each line.

a) \(x - 4y = 12 - 4y\)  

b) \(2x - 3y = 7\)
Finding Intercepts
- The x-intercept is of the form \((__, __)\). To find it, set \(y = 0\) and solve for \(x\).
- The y-intercept is of the form \((__, __)\). To find it, set \(x = 0\) and solve for \(y\).

Ex 2
Find the intercepts. Then graph by using the intercepts, if possible, and a third as a check.

a) \(x + y = 4\)

b) \(5x - 4y = 20\)

c) \(5y = 15x\)

d) \(\frac{1}{2}y + \frac{1}{3}x = 1\)

Standard Form of a Line
Any equation of the form \(Ax + By = C\), where \(A\), \(B\), and \(C\) are real numbers and \(A\) and \(B\) are not both 0, is a **linear equation in standard form**. The graph is a straight line.

Ex 3
Determine whether the equation is linear. Find the slope of any non-vertical lines.

a) \(3x + 5y + 15 = 0\)

b) \(2y - 30 = 0\)

c) \(2x + 4f(x) = 8\)

d) \(y(3 - x) = 2\)

e) \(g(x) - x^3 = 0\)

\[f) \frac{1}{2}(x - 4) = y\]

g) \(y = \frac{10}{x}\)

**Sec 2.5: Other Equations of Lines**

Ex 1
What is the slope of a line that passes through \((x_1, y_1)\) and \((x, y)\)? Multiply both sides be denom. to obtain point-slope form.

Point-Slope Form
Any equation of the form \(y - y_1 = m(x - x_1)\) is said to be written in **point-slope** form and has a graph that is a straight line.
The slope is \(m\). The line passes through \((x_1, y_1)\).

Do #1 and #2 from Points-Slope Equation of the Line Handout.

Parallel and Perpendicular Lines
Two lines are parallel if they have the same slope or if they are both vertical.

Two lines are perpendicular if the product of their slopes is \(-1\) or if one line is vertical and the other line is horizontal.

Do #3-#5 from handout.

Ex 2
If time permits
Without graphing, determine if the graphs of the pair of equations are parallel, perpendicular, or neither.
\(2x - 5y = -3\)
\(2x + 5y = 4\)
Ex 3  If time permits (# 48) In 1994, the life expectancy of males was 72.4 years. In 2004, it was 75.2 years. Let $E(t)$ represent life expectancy and $t$ the number of years since 1990. Assume that a constant rate of change exists for the model.

a) Find a linear function that fits the data.

b) Use the function of part (a) to predict the life expectancy of males in 2012. 77.44 years

Must memorize point-slope and slope-intercept form for quizzes and exams.

**Sec 2.6: The Algebra of Functions**

Let $f$ and $g$ be functions where $x$ is in the domain of both functions. Then

1) $(f + g)(x) = f(x) + g(x)$

2) $(f - g)(x) = f(x) - g(x)$

3) $(f \cdot g)(x) = (f)(g)(x)$

4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$.

**Ex 1** Let $f(x) = -2x + 3$ and $g(x) = x^2 - 5$. Find each.

a) $f(4) + g(4)$

b) $g(-3)/f(-3)$

c) $(g - f)(x)$

d) $(fg)(x)$

e) $\left(\frac{g}{f}\right)(x)$

**Ex 2** Let $G(x) = x^2 - 2x - 1$. Find $G(x + h)$.

Determining the Domain

The domain of $f + g$, $f - g$, or $fg$ is the set of all values common to the domains of $f$ and $g$.

The domain of $\frac{f}{g}$ is the set of all values common to the domains of $f$ and $g$, excluding any values for which $g(x) = 0$.

**Ex 3** For the pair of functions, determine the domain of the sum, difference, and product of the two functions. $f(x) = x^3 + 1$

$g(x) = \frac{5}{x}$

**Ex 4** Determine the domain of $\frac{f}{g}$ where $f(x) = x + 2$ and $g(x) = x^2 - 4$.

**Ex 5** Let $f(x) = x^2 - 1$. Find each.

$f(2)$

$f(-3)$

$f(x) = 11$

$f(t + 1)$

$2f(x)$

$\frac{1}{2}f(x - 1) + f(x) - 3f(-1)$