

Objective 1 – The real number line:

*****No calculator allowed!**

Let's begin with vocabulary regarding subsets of the set of **real numbers**. The subset that you more than likely began your mathematical career with is the set of **natural, or counting, numbers**: $\{1, 2, 3, \dots\}$. Then, we threw 0 in the mix, forming the set of **whole numbers**: $\{0, 1, 2, 3, \dots\}$. For those whose glass is half-empty, we invited some negatives to the party, forming the set of **integers**: $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$. Basically, a **rational number** is any number that can be put into strict fraction form, which includes integers and any positive or negative terminating or repeating decimal, proper or improper fraction, and mixed number. An **irrational number** is a number that is not rational (i.e. it can't be represented as a fraction, like π , $\sqrt{2}$, e , ϕ , $-9.101001\dots$ to name a few). Penultimately, the real numbers are the union of the set of rational numbers and the set of irrational numbers. BTW: Penultimately means "second to last," and that is essentially a forecast, as there exists one more set of numbers to be reviewed, namely, the **complex numbers**, wherein you'll find the subset of imaginary numbers (see objective 17) .

Give the set, algebraic, and interval notation for each set of real numbers described. Graph each set on a number line.

- 1) No more than π .
- 2) At least -6.5 .
- 3) Less than $-3\frac{13}{16}$.
- 4) All real numbers greater than $-\sqrt[3]{18}$ and less than or equal to e .

Objective 2 – Solve absolute value equations and inequalities:

If $|A| = k$, then $A = k$ or $A = -k$. If $|A| > k$, then $A > k$ **or** $A < -k$. If $|A| \leq k$, then $A \geq -k$ **and** $A \leq k$, which is equivalent to $-k \leq A \leq k$. Be aware, $k \geq 0$ for all three of these absolute value relationships. Special cases occur when $k < 0$, yielding either "no solution" or "all real numbers."

Solve the following:

5) $\left|5 - \frac{2}{3}x\right| = 7$ 6) $|(3x - 2)6| \geq 42$ 7) $|1 - 10x| < 6$

Objective 3 – Translate English phrases to mathematical expressions:

Translate each phrase into a mathematical expression.

- 8) Forty-four **subtracted from** twice a number.
- 9) Six thousand twenty-seven **less than** five times a number.
- 10) The **product of** ninety and the **quotient of** two consecutive odd integers.

Objective 4 – Operations on rational numbers:

Be able to add, subtract, multiply, and divide any two rational numbers.

11) Subtract $8\frac{7}{12}$ from $12\frac{3}{8}$.

12) Divide: $\frac{-4\frac{2}{3} \div 12}{-5\frac{3}{4} - 6\frac{7}{8}}$

Note: The answer to exercise 12 isn't pretty, but then again, life's not always pretty.

Objective 5 – Distribute and combine like terms:

13) Simplify: $6m(4m^2 - 3mn + 5n^2) - 8 - (25m^3 + 7m^2n - 10mn^2) - (-4)$

Objective 6 – Fundamentals of geometry:

You should have committed to memory each of the following facts:

- 14) What it means to find the perimeter of a polygon.
- 15) The area of a rectangle .
- 16) The number of degrees every triangle's three angles sum to.
- 17) The area of a triangle.
- 18) What a trapezoid looks like.
- 19) The area of a circle.
- 20) The circumference of a circle.
- 21) The volume of a rectangular prism (a box).
- 22) The volume of a cylinder (a can).
- 23) What each type of triangle (acute, right, obtuse, scalene, isosceles, equilateral) looks like.

Objective 7 – Solve each of the five types of fundamental percent problems:

- 24) 65% of a number is 2080. Find the number.
- 25) 44 is what percent of 80?
- 26) If the tax rate is 7%, what would tax be on a new car that costs \$18,800?
- 27) Last year, a pound of lean ground beef cost \$4.50. This year, it costs \$6.00. What is the percent of increase?
- 28) In 2003, Enron stock peaked at \$90/share . In 2004, it peaked at \$30/share. What is the percent of decrease?

Objective 8 – Conversion:

Be able to convert back and forth between any two of the following: **Decimal, fraction, percent**. At any level of mathematics, it goes without saying that fractions must always be reduced to lowest terms, and one should never write a fraction with a decimal part in either its numerator or its denominator.

- a) D to F: Write the decimal as you would say it aloud, in English form, then write the fraction equivalent.
 - b) F to D: Divide the denominator of the given fraction into the numerator.
 - c) D to P: Slide the decimal point two places right and tack on a percent symbol.
 - d) P to D: Remove the percent symbol and then slide the decimal point two places left.
 - e) F to P: Do F to D and then D to P, or... set the fraction equal to P/100 and solve the proportion for P.
 - f) P to F: Do P to D and then D to P, or... drop the percent and write what's left over 100.
- 29) Convert .075 to a fraction.
 - 30) Convert 250% to a fraction.
 - 31) Convert 5% to a decimal.
 - 32) Convert 7/11 to a decimal.
 - 33) Convert 0.003 to a percent.
 - 34) Convert 2/3 to a percent.

Objective 9 – Order of operations:

Be able to apply the order of operations to either compute or evaluate an expression. The order of operations is given by PEMDAS: **1) P**arentheses and other grouping symbols, **2) E**xponents and radicals, **3) all M**ultiplications and **D**ivisions in the order they appear from left to right, and **4) all A**dditions and **S**ubtractions in the order they appear from left to right.

- 35) Compute: $8 - 3^2 + \sqrt{49} + 36 \div 4 \times 3$
- 36) Evaluate: $-w^2 + \frac{q(p-3w)}{w-q}$; $p = -2$, $q = 3$, $w = -4$

Objective 10 – Scientific notation:

First and foremost, know that a number is in scientific notation when it looks like this: $A \times 10^k$, where $1 \leq A < 10$. Secondly, add exponents when multiplying powers of 10, and subtract them when dividing.

$$37) \text{ Compute: } (6 \times 10^{-12})(9 \times 10^{19}) \qquad 38) \text{ Compute: } \frac{14,400,000}{0.00000000016}$$

Objective 11 – The solving process:

If you **multiply or divide every term in an inequality by a negative number**, you must **flip the inequality** sign(s). Further, if the variable term drops out completely, then what's left will be either an **identity**, in which case the solution set is **"all real numbers,"** or a **contradiction**, in which case the solution set is **"no real numbers."** With all that on the table, the steps to solving a single variable equation or inequality are **1)** Clear parentheses using the distributive property, clear fractions using the LCD, and/or clear decimals using a power of 10, **2)** Combine like terms, **3)** Get the variable terms on one side and non-variable terms on the opposite side, and **4)** Divide both sides by the coefficient of the variable term. Whenever there is more than one variable term and they can't be combined, factor out the variable and then divide.

$$39) \text{ Solve: } 24 - 2(5x - 3) \geq 4x + 7(4 - 2x) \qquad 40) \text{ Solve for } L: \frac{3}{J} + \frac{2}{K} = \frac{1}{L}$$

Objective 12 – Rules of exponents:

These rules apply whenever a , A , and b are real, m , n , and k are integers, and denominators aren't 0.

$$\text{Product Rule: } (b^m)(b^n) = b^{m+n} \qquad \text{Quotient Rule: } \frac{b^m}{b^n} = b^{m-n} \qquad \text{Negative Exponent Rule: } b^{-m} = \frac{1}{b^m}$$

$$\text{Power Rules: } (b^m)^n = b^{mn} \qquad (a^m b^n)^k = a^{km} b^{kn} \qquad \left(\frac{a^m}{b^n}\right)^k = \frac{a^{km}}{b^{kn}} \qquad \left(\frac{a^m}{b^n}\right)^{-k} = \frac{b^{kn}}{a^{km}}$$

$$\text{More Rules: } A^{-1} = \frac{1}{A} \qquad A^0 = 1 \text{ (provided } A \neq 0) \qquad b^{m/n} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m \text{ (provided } n \geq 2 \text{ \& } \sqrt[n]{b} \text{ exists)}$$

$$\text{Be aware: } (A + B)^2 \neq A^2 + B^2 \qquad \text{Rather: } (A + B)^2 = A^2 + 2AB + B^2$$

Trickonometry: $-A^2$ is always negative, regardless of what real number A is! That is, $-A^2 = -(A \cdot A)$
If you wish to square a negative real number, then it must be put in parentheses first!
That is, if you wish to square -6 , then think $(-6)^2$, to get the answer: 36
Whatever real number you square, it will always be positive! That is, $(-A)^2 = (-A)(-A)$

$$\text{Simplify, and write your answer without negative exponents: } 41) (5x^{-4}y^{-3}z^2)^{-2} \qquad 42) \frac{3^{80}u^{11}v w^{-7}}{3^{84}u^8v^{-2}w}$$

Objective 13 – Polynomials:

When working with polynomials, factoring skills are of utmost importance. Here are some factoring facts that should be committed to memory:

Always look to remove a GCF first! Absent a GCF, a sum of two squares $(A^2 + B^2)$ cannot be factored!

A difference of two squares: $A^2 - B^2 = (A + B)(A - B)$

A perfect square trinomial: $A^2 + 2AB + B^2 = (A + B)^2$ $A^2 - 2AB + B^2 = (A - B)^2$

A sum & difference of two cubes: $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$ $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

Whenever you must factor an expression with four or more terms, consider "factor by grouping."

Just to get up to speed on some of the common vocabulary used in association with polynomials, consider this example of a **six term, 7th degree** polynomial expression: $-9x^7 - x^5 + \frac{4}{7}x^3 + x^2 - 20x + 17$

Its **leading coefficient** is -9 . The coefficient on the second degree term is 1 while the coefficient on the fifth degree term is -1 . The coefficient of the zeroth degree term is 17, but this is usually referred to as the **constant**. Notice that the coefficient of both the sixth and fourth degree terms is 0.

Though not officially a polynomial, the following expression (and the like) is usually always part of a discussion involving polynomials: $-16t^3u^7v + 10tuv^{10}$

To get the degree of each term, you must "add the powers" on each variable in the term. Thus, the degree of the first term is $3+7+1=11$ and the degree of the second term is $1+1+10=12$. The highest degree of the individual terms is the degree of the expression. Thus, this is a **12th-degree expression**.

43) Simplify: $(8c - 5d)^2$ 44) Solve: $7x^3 + 2x^2 - 28x - 8 = 0$

45) Divide: $\frac{9x^5y^7 - 15x^6y^3 + 5x^2y - 35xy}{5x^2y}$ 46) Divide: $\frac{12x^3 - 11x^2 - 13x + 6}{4x - 5}$

Objective 14 – Rational expressions:

A rational expression is basically the quotient of two polynomials. Recall there are 2 ways to simplify complex fractions, like in #47. One involves simplifying the numerator, then simplifying the denominator, and then dividing the answers. The other method involves first multiplying the numerator and the denominator by the LCD of all of the simple fractions.

47) Simplify: $\frac{\frac{1}{3} - \frac{1}{x}}{\frac{1}{6} - \frac{1}{2x}}$ 48) Divide: $\frac{y^2 - 4}{y^2 + 5y + 6} \div \frac{y^2 + y - 6}{y^2 - 9}$ 49) Solve: $\frac{6}{x+2} - 4 = \frac{x+20}{3x+6}$

Objective 15 – Radical expressions:

Note that: $b^{m/n} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$ (provided $n \geq 2$ & $\sqrt[n]{b}$ exists)

50) Simplify: $\sqrt{44} - \sqrt{27} - 5\sqrt{12} - \sqrt{11}$

51) Rationalize the denominator: $\frac{4}{\sqrt{6} - 2}$

52) Solve: $\sqrt{5-4x} = x+10$

53) Solve: $(2x+1)^{3/2} = 27$

Objective 16 – Solving Polynomial (most often quadratic) Equations:

Be sure to commit to memory the quadratic formula: $ax^2 + bx + c = 0$ implies $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

54) Solve by factoring: $10x^3 = 14x^2 + 12x$

55) Solve by extracting square roots: $9x^2 + 17 = 18$

56) Solve by completing the square: $x^2 + 14x = 11$

57) Solve by using the quadratic formula: $3(x^2 + 1) = 8x$

Objective 17 – Operations on complex numbers:

A complex number is any number in the form $a + bi$, where a & b are both real numbers and $i = \sqrt{-1}$.

58) Simplify: $(-6 + 13i) - (5 - 3i)$

59) Rationalize the denominator: $\frac{6}{3-2i}$

Objective 18 – Variation:

Be able to solve fundamental variation problems involving **direct, inverse, and joint variation**. Solving these sorts of problems usually requires three steps: **1)** Write the equation of variation (see the notes that follow), **2)** Substituting known quantities to determine the “**constant of proportionality k ,**” and **3)** Answering the question posed.

When two quantities are directly proportional, it can be said that they vary directly, and vice-versa. When A varies *directly as* B , it means that their quotient is constant. The equation of variation is $A = kB$

When two quantities are inversely proportional, it can be said that they vary inversely, and vice-versa. When A varies *inversely as* B , it means that their product is constant. The equation of variation is $AB = k$

When two quantities are jointly proportional, it can be said that they vary jointly, and vice-versa. Joint variation implies a relationship between three or more quantities, where one quantity is *directly proportional* to each of the others taken one at a time. When A varies *jointly as* B and C , the equation of variation is $A = kBC$

60) If G and H are **directly** proportional, and G equals 8 when H is 30, find H when G is 20.

61) If G and H are **inversely** proportional, and G equals 25 when H is 40, find G when H is -30 .

62) If F varies **jointly** with the square root of G and half of H , and F is 3 when G is 9 and H is 50, find H when G is 49 and F is 12.

Objective 19 – Functions:

Be able to evaluate, graph, and give the domain, range, and intercepts of a basic linear, quadratic, rational, root, or absolute value function. The domain is the set of all x-values for which the function is defined. The range is the set of all y-values for which the function is defined. You can always get an x-intercept, if one or more exist, by substituting 0 for y, or f(x). You can always get the y-intercept, also known as f(0), if one exists, by substituting 0 for x.

The domain of every linear, quadratic, and absolute function is “all real numbers” (a.k.a. $(-\infty, \infty)$ or \mathbb{R}).

To get the domain of a root function such as $f(x) = \sqrt{A}$, solve $A \geq 0$ for x .

To get the domain of a rational function such as $f(x) = \frac{A}{B}$, set $B = 0$ and solve for x to determine the real numbers that **are not** in the domain of the given function.

63) Given $f(x) = |4 - 12x|$, find the domain, range, intercepts, and $f(-3)$.

64) Given $f(x) = \frac{x+7}{x+5}$, find the domain, intercepts, and $f(3)$.

Graph each of the following functions:

65) Linear: $f(x) = \frac{3}{10}x - 1$

66) Quadratic: $f(x) = x^2 - 10x - 24$

67) Rational: $f(x) = \frac{1}{x-6}$

68) Root: $f(x) = 3 + \sqrt{x+4}$

69) Absolute Value: $f(x) = |x-8| - 2$

Objective 20 – Operations on functions:

You should be able to add, subtract, multiply, and divide any two functions. Note here that

a) $[f + g](x) = f(x) + g(x)$

b) $[f - g](x) = f(x) - g(x)$

c) $[fg](x) = f(x)g(x)$

d) $\left[\frac{f}{g}\right](x) = f(x) \div g(x)$, provided $g(x) \neq 0$.

70) Given $f(x) = \frac{3}{x-2}$ & $g(x) = \frac{5}{x+1}$, find $[f - g](x)$.

71) Given $f(x) = 3x + 2$ & $g(x) = 9x^2 - 6x + 4$, find $[fg](x)$.

Objective 22 – Linear Models:

You should be able to determine the equation of a line given sufficient information. Recall that slope-intercept form is given by $y = mx + b$, and standard form is $Ax + By = C$ where A , B , and C are integers (if possible) with A positive. If the line is horizontal, then its slope is 0 and standard form is $y = c$, where c is some real number constant. If the line is vertical, then its slope is undefined and standard form is $x = k$, where k is some real number constant. Parallel lines have equal slopes, and perpendicular lines have slopes which are “negative reciprocals” of each other (i.e. their product is -1). A useful tool for finding equations is point-slope form, given by $y - y_1 = m(x - x_1)$.

72) Find the equation of the line thru $(-4, 6)$ and perpendicular to $y = \frac{2}{5}x + 19$.

73) Find the equation of the line thru $(16, -27)$ and parallel to $6x + 23 = 20$.

74) Let $x =$ age in years and $y =$ value in dollars and assume “delayed” depreciation on a new RV from the moment one drives off the sales lot is linear. Find the mathematical model (in slope-intercept form) for delayed depreciation of an RV that was valued at \$50,000 when it was 6 years old, and at 11 years, valued at \$20,000.

Objective 21 – Finding the inverse of a function:

To find the inverse of a function, denoted $f^{-1}(x)$, do the following:

- a) Change $f(x)$ to y .
- b) Interchange x and y .
- c) Solve for the new y .
- d) Change the y to $f^{-1}(x)$, which is proper inverse function notation.

75) Identify the slope and y-intercept of the following linear function: $f(x) = \frac{6-8x}{3}$. Then find $f^{-1}(x)$.

Finally, graph f , f^{-1} , and the most famous line in mathematics, $y = x$, in a rectangular plane.

You should see that the graphs of f & f^{-1} are symmetric about the line $y = x$.

Objective 23 – Systems:

You should be comfortable solving a system by either the substitution method or by the elimination method (sometimes referred to as the addition method). There are **three kinds of 2×2 linear systems:** **1)** The system has exactly one solution, in which case it is referred to as a consistent system, and the geometry involves two intersecting lines, **2)** The system has no solution, in which case it is referred to as an inconsistent system, and the geometry involves two parallel lines, and **3)** The system has infinitely many solutions, in which case it is referred to as a dependent system, and the geometry involves two coinciding lines. **Systems of inequalities can only be solved by graphing and shading!**

76) Solve:
$$\begin{aligned} 6x - 16y &= -10 \\ 9x - 24y &= -5 \end{aligned}$$

77) Solve:
$$\begin{aligned} x &= 7y + 14 \\ \frac{3}{7}x + 3(5 - y) &= 21 \end{aligned}$$

78) Solve by graphing:
$$\begin{aligned} y &< -\frac{11}{3}x + 7 \\ 5x - 8y &\leq 16 \end{aligned}$$

79) Determine the ordered triple (x, y, z) that satisfies: $3x - 2y + 5z = 2$

$$4x - 7y - z = 19$$

$$5x - 6y + 4z = 13$$

80) Solve by substitution to find where the graphs intersect: $x - 3y = 8$ (a line)
 $3x - 2y^2 = 8y + 3$ (a sideways parabola)

81) Solve by elimination to find where the graphs intersect: $9x^2 + 4y^2 = 36$ (an ellipse)
 $25x^2 - 16y^2 = 405$ (a hyperbola)

Objective 24 – Word problems:

You'll need a 6-pak for these, as in a 6-step process: **1)** Define the unknown(s), **2)** Draw a picture, set up a chart, or recall a formula if necessary, **3)** Construct the equation(s), **4)** Solve the equation(s), **5)** Answer the question, and **6)** Do at least an "Is my answer reasonable?" check, if not a knock-down, drag-out formal check. Now, there's a ton of word problems we could practice here, but in the interest of review, we'll limit it to a quadratic uniform motion problem a two "classic" mixture problems.

82) A motorboat heads upstream a distance of 24 miles on a river whose current is running at 3 miles per hour. The trip up and back takes 6 hours. Assuming that the motorboat maintained a constant speed relative to the water, what was its speed?

83) A coffee maker wants to market 100 pounds of a new blend of coffee that sells for \$3.90 per pound by mixing two coffees that sell for \$2.75 and \$5 per pound, respectively. What amounts of each coffee should be blended to obtain the desired mixture?

84) A 20-pound bag of low-grade cement mix contains 25% cement and 75% sand. How much pure cement must be added to produce a cement mix that is 40% cement?

Objective 25 – Coordinate geometry:

Given any two points in the rectangular coordinate plane, you should be able to determine their midpoint, the distance between them, and the slope of the line that they determine.

Given points $P(x_1, y_1)$ and $Q(x_2, y_2)$, the midpoint formula is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, the distance formula is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, and the slope formula is $\frac{y_2 - y_1}{x_2 - x_1}$.

85) Find the midpoint, distance, and slope values for the points $(8, 3)$ and $(0, -1)$.

86) Thank you choosing not to "eighty-six" this critical practice.